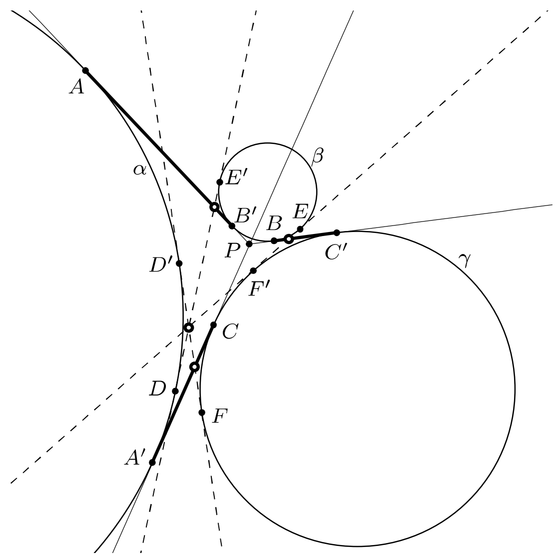

AB', BC', CA' all pass through P, so that $AB' = BC' + CA'$. Observe that because $CA' = FD'$, we also have $AB' = BC' + FD'$ even though the lines AB', BC', FD' are not concurrent. Somehow one must discover how to make use of the hypothesis that the lines AB', BC', CA' are concurrent (which implies that $AB' = BC' + CA'$ and, therefore, that $DE' = EF' + FD'$). The conjecture appears even harder to prove; for the conjecture one would be given only $AB' = BC' + CA'$ with some assignment of labels, and would have to describe how to correctly select a subset of three concurrent lines from the given set of six common internal tangent lines.

No solutions have been received; the problem remains open.

3749. [2012 : 195, 197] *Proposed by Yakub N. Aliyev, Qafqaz University, Khyrdalan, Azerbaijan.*

Let D and E be arbitrary points on the sides BC and AC of a triangle ABC. Prove that

$$\sqrt{[ADE]} + \sqrt{[BDE]} \leq \sqrt{[ABC]},$$

where $[XYZ]$ denotes the area of triangle XYZ.

Solution by Arkady Alt, San Jose, CA, USA.

Let $t = \dfrac{BD}{BC}$, $s = \dfrac{CE}{CA}$, and $F = [ABC]$. Then $t, s \in [0,1]$,

$$[BDE] = Fst,$$

and

$$[ADE] = F\,(1-s)\,(1-t)\,.$$

Hence the claimed inequality is equivalent to the inequality

$$\sqrt{(1-s)(1-t)}+\sqrt{st}\leq 1,$$

which follows immediately from the Cauchy-Schwarz inequality applied to the pairs $\left\langle\sqrt{1-s},\sqrt{s}\right\rangle$ and $\left\langle\sqrt{1-t},\sqrt{t}\right\rangle$.

Also solved by MIGUEL AMENGUAL COVAS, Cala Figuera, Mallorca, Spain; AN-ANDUUD Problem Solving Group, Ulaanbaatar, Mongolia; GEORGE APOSTOLOPOULOS, Messolonghi, Greece; ŠEFKET ARSLANAGIĆ, University of Sarajevo, Sarajevo, Bosnia and Herzegovina (2 solutions); ROY BARBARA, Lebanese University, Fanar, Lebanon; MICHEL BATAILLE, Rouen, France; CHIP CURTIS, Missouri Southern State University, Joplin, MO, USA; PRITHWIJIT DE, Homi Bhabha Centre for Science Education, Mumbai, India; NERMIN HODŽIĆ, Dobošnica, Bosnia and Herzegovina and SALEM MALIKIĆ, student, Simon Fraser University, Burnaby, BC; DIMITRIOS KOUKAKIS, Kilkis, Greece; KEE-WAI LAU, Hong Kong, China; CAO MINH QUANG, Nguyen Binh Khiem High School, Vinh Long, Vietnam; CRISTÓBAL SÁNCHEZ–RUBIO, I.B. Penyagolosa, Castellón, Spain; ALBERT STADLER, Herrliberg, Switzerland; IRINA STALLION, Southeast Missouri State University, Cape Girardeau, MO, USA; EDMUND SWYLAN, Riga, Latvia; ITACHI UCHIHA, Hong Kong, China; DANIEL VĂCARU, Piteşti, Romania; HAOHAO WANG and JERZY WOJDYLO, Southeast Missouri State University, Cape Girardeau, Missouri, USA; PETER Y. WOO, Biola University, La Mirada, CA, USA; TITU ZVONARU, Comăneşti, Romania; and the proposer.

3750. [2012 : 195, 197] *Proposed by Michel Bataille, Rouen, France.*

Let $T_k = 1+2+\cdots+k$ be the k^{th} triangular number. Find all positive integers m, n such that $T_m = 2T_n$.

Composite of submitted solutions.

The equation is equivalent to $m(m+1) = 2n(n+1)$, which in turn becomes $x^2-2y^2=-1$ when $x = 2m+1$ and $y = 2n+1$. The positive solutions $(x,y) = (x_k,y_k)$ of the Pellian equation $x^2-2y^2=-1$ are all odd and given by

$$x_k + y_k\sqrt{2} = (1+\sqrt{2})(3+2\sqrt{2})^k$$

for $k = 0,1,2,\ldots$.

There are various ways of describing these solutions:

1. $(x_0,y_0) = (1,1)$, and $(x_{k+1},y_{k+1}) = (3x_k+4y_k, 2x_k+3y_k)$;

2. $(x_0,y_0) = (1,1)$, $(x_1,y_1) = (7,5)$, and

$$(x_{k+2},y_{k+2}) = 6(x_{k+1},y_{k+1}) - (x_k,y_k);$$

3.

$$x_k = \frac{1}{2}\left[(1+\sqrt{2})^{2k+1}+(1-\sqrt{2})^{2k+1}\right],$$
$$y_k = \frac{1}{2\sqrt{2}}\left[(1+\sqrt{2})^{2k+1}-(1-\sqrt{2})^{2k+1}\right];$$